Momentum, Heat and Mass Transfer

Mass Transfer

The rate of diffusion of a constituent A in a mixture is proportion to its concentration gradient according to Fick's Law of diffusion

$$N_A = -D\,\frac{dC_A}{dy}$$

Where :

N_A : is the molar rate of diffusion of constituent A per unit area , (Kmol / m^2 .s)
C_A :is the molar concentration of constituent A , (Kmol/m^3)
D : is the mass diffusivity , (m^2/sec.)

$\frac{\mu}{\rho}$ ***is called the kinematic viscosity*** (transfer of the momentum)

$\frac{k}{\rho c_p}$ ***is called the thermal diffusivity*** D_H (transfer of the heat)

D is called the diffusivity (transfer of the mass)

The essential similarity between the three processes is that the rates of transfer of momentum, heat, and mass are all proportional to the concentration gradients of these quantities.

All have a dimensions (L^2 /time) (m^2/s)

Momentum, Heat and Mass Transfer

Viscosity

Consider the flow of a gas parallel to a solid surface and the movement of molecules at right angles to this direction through a plane a-a of unit area, parallel to the surface and sufficiently close to it to be within the laminar sublayer as shown in figure

During an interval of time **dt**, molecules with an average velocity $i_1 u_m$ in the Y-direction will pass through the plane (where u_m is the root mean square velocity and $\boldsymbol{i_1}$ is some fraction of u_m, depending on the actual distribution of velocities).

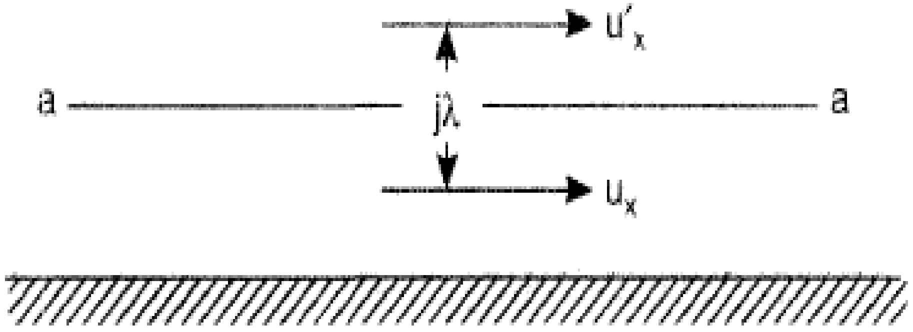


Figure (1) Transfer of momentum near a surface

$$\boldsymbol{u_m = \sqrt{\frac{u_1+u_2+\cdots.u_n}{no.of\ molecules}} = \sqrt{\frac{\sum u_n}{no.of\ molecules}}}$$

$$\boldsymbol{i_1 = \frac{avarage\ velocity}{root\ mean\ velocity}}$$

If all these molecules can be considered as having the same component of velocity in the Y -direction, molecules from a volume = $\boldsymbol{i_1 u_m}$**(m/s) (1 m^2) dt** will pass through the plane in time dt.

If **N** is the numerical concentration of molecules close to the surface, = $\frac{no.of\ molecules}{volume}$

the number of molecules passing = $i_1 u_m\ N.dt$

Thus, ***the rate of passage of molecules*** = $\frac{i_1 . u_m . \text{(m/s) N . (1) . dt}}{dt} = i_1\ u_m\ N$ (mole/(m^2 s)

These molecules have a mean velocity u_x (say) in the X-direction.

Thus the rate at which momentum is transferred across the plane away from the surface $= i_1 . N . u_m . m . u_x$

where m is the mass of each molecule.

By similar reasoning there must be an equivalent stream of molecules also passing through the plane(a-a) but in the opposite direction; otherwise there would be a resultant flow perpendicular to the surface.

If this other stream of molecules has originated at a distance $\boldsymbol{j\lambda}$ from the previous ones, and the mean component of their velocities in the X-direction is u_x'

where $\boldsymbol{\lambda}$ is the mean free path of the molecules and j is some fraction of the order of unity then:

The net rate of transfer of momentum away from the surface

$= i_1 N\, u_m\, m(u_x - u_x')$

The gradient of the velocity with respect to the Y-direction $\frac{du_x}{dy} = \frac{(u_x' - u_x)}{j\lambda}$

Since λ is small

Thus the rate of transfer of momentum per unit area which can be written as:

$R_y = -\, i_1\, \boldsymbol{N}.\boldsymbol{u_m}\, .\boldsymbol{m}\;\, j\lambda$

(the density of the fluid ρ =N . m)

$$= -\boldsymbol{i_1}\boldsymbol{u_m}\boldsymbol{\rho} j\lambda \frac{du_m}{dy}$$

And $$R_y = -\mu \frac{du_x}{dy}$$

$$\frac{\mu}{\rho} = \boldsymbol{i_1 u_m}\, j\lambda$$

The value of product (i.j) = $\frac{1}{2}$ from statistical treatment of the velocities of the molecules

$$\frac{\mu}{\rho} = \frac{1}{2}\boldsymbol{u_m}\lambda$$

It is now possible to give a physical interpretation to the Reynolds number:

$$R_e = \frac{\rho u d}{\mu} = ud\,\frac{\mathbf{2}}{\boldsymbol{u_m\lambda}} = 2\,\frac{u}{u_m}\cdot\frac{d}{\lambda}$$

Re is proportional to the product of the ratio of the flow velocity to the molecular velocity and the ratio of the characteristic linear dimension of the system to the mean free path of the molecules.

From kinetic theory:

$u_m = \sqrt{\frac{8RT}{\pi\, m}}$ $\lambda = \sqrt{\frac{RT}{2\pi m}}$ for gases only . m= molecular weight

So, viscosity of a gas would be a function of temperature but not of pressure.

Units $\mu = poise$ (1 g·cm^{-1}·s^{-1}) T=°K R=$\frac{Newton\quad m}{K_{mol}\quad °\mathrm{K}}$

Thermal conductivity :-

The same figure (1) where is a transfer of temperature gradient in y- direction. The rate of passage of molecule through the unit plane (a-a) $=\frac{i_2 u_m N.1dt}{dt} = i_2 u_m N$

i_2=is some fraction of order of unity .

If the temperature difference between two points situated a distance (j. λ) a part is $(\theta - \theta')$.

The net heat transfer as one molecule pass=$1.c_m(\theta - \theta')$

Where =c_m=is the heat capacity per 1 molecule .

The temperature gradient =$\frac{d\theta}{dy}=\frac{\theta-\theta}{\lambda j}$

The net rate of heat transfer per unit area =q_y = - $i_2 u_m$. N c_m. $\lambda j \frac{d\theta}{dy}$

Since Nc_m= ρc_v specific heat per unit volume of fluid.

q_y= - $i_2 . j . u_m . c_v . \rho . \lambda \frac{d\theta}{dy}$ but , q_y=-k$\frac{d\theta}{dy}$

-k$\frac{d\theta}{dy}$ = - $i_2 . j . u_m . c_v . \rho . \lambda \frac{d\theta}{dy}$

Thus thermal diffusivity:-

$$\frac{k}{c_p.\rho} = i_2.j.u_m.\lambda\frac{c_v}{c_p} \quad (\div c_p) \qquad D_H = \frac{k}{c_p.\rho} \quad thermal\ \textbf{diffusivity}(m^2/\mathrm{s})$$

From statistical calculation $i_2.j = (\frac{9\,\gamma - 5}{8})$ where $\gamma = \frac{c_p}{c_v}$

$$\frac{K}{c_p.\rho} = (\frac{q\,\gamma\_5}{8\,\gamma})\ .u_m.\ \lambda$$

The Prandtl number Pr is defined as the ratio of the kinematic viscosity to the thermal diffusivity.

$$p_r = \frac{\mu/\rho}{\frac{K}{\rho\ c_\rho}} \quad = \frac{c_p.\mu}{K} \quad = \frac{\frac{1}{2}u_m.\lambda}{(\frac{q\,\gamma\_5}{8\,\gamma})\ .u_m.\ \lambda} = \frac{4\ \ \gamma}{9\gamma - 5}$$ for molecular diffusion.

$$\boldsymbol{R_e} = \frac{\mathbf{2u}}{\boldsymbol{u_m}}.\frac{\boldsymbol{d}}{\boldsymbol{\lambda}} \qquad\qquad \boldsymbol{P_r} = \frac{\mathbf{4}\ \ \boldsymbol{\gamma}}{\mathbf{9}\boldsymbol{\gamma}\mathbf{-5}}$$

Diffusivity :-

Considering the diffusion, in the Y-direction, of one constituent A of a mixture across the plane a-a, If the numerical concentration is C_A on one side and C'_A in the other side which is operated by a distance

(jλ) .

Net rate passage of molecules=$i_3 u_m(C_A - C'_A)$

i_3= is a fraction of the order of unity .

Rate of mass transfer per unit area =$i_3 u_m$ $(C_A - C'_A)$.m

Concentration gradient of A in Y- direction =$\frac{dc_a}{dy} = \frac{C'_A - C_A}{j.\lambda}$

The rate of mass transfer per unit area =- $i_3 . u_m$.j.$\lambda \frac{dC_a}{dy}$

$N_A = -D\frac{dC_A}{dy}$

D $= i_3$.j.λ . u_m

There is, however, no satisfactory evaluation of the product i_3.j.

The ratio of the kinematic viscosity to the diffusivity is the Schmidt number, *Sc,* where;

$$s_c = \frac{\mu/\rho}{D} = \frac{\mu}{\rho\, D} = \frac{i_1 .\mathrm{j}.\lambda\ .u_m}{i_3 .\mathrm{j}.\lambda\ .u_m}$$

$\frac{\mu}{\rho} = i_1$.j.λ . u_m kinematic viscosity

$\frac{K}{c_p\ \rho} = i_2$.j.$\lambda$. u_m thermal diffusivity

D$= i_3$.j.λ . u_m diffusivity

$\frac{\mu}{\rho}$ Or $\frac{K}{c_p\ \rho}$ Or D α u_m λ

It is thus seen that the kinematic viscosity, the thermal diffusivity, and the diffusivity for mass transfer are all proportional to the product of the ***mean free path*** and the ***root mean square velocity*** of the molecules, and that the expressions for the transfer of momentum, heat, and mass are of the same form.

For liquids the same qualitative forms of relationships exist, but it is not possible to express the physical properties of the liquids in terms of molecular velocities and distances.

EDDY TRANSFER (Turbulent flow)

A fluid in turbulent flow is characterised by the presence of circulating or eddy currents, and these are responsible for fluid mixing which, in turn, gives rise to momentum, heat or mass transfer when there is an appropriate gradient of the "property“.

PRANDTL and TAYLOR both developed the concept of a *mixing length* (λ_E)as a **measure of the distance** which an element of fluid must travel before it loses its original identity and becomes fully assimilated by the fluid in its new position.
The mixing length is analogous in concept to the *mean free path* of gas molecules which, according to the kinetic theory is the mean distance a molecule travels before it collides with another molecule and loses its original identity.

In turbulent flow over a surface, a velocity gradient, and hence a momentum gradient, exists within the fluid. Any random movement perpendicular to the surface gives rise to a momentum transfer. Elements of fluid with high velocities are brought from remote regions towards the surface and change places with slower moving fluid elements.

For turbulent flow the velocity profile with mean flow in Y- direction at point (1) the mean velocity is u_1, a portion of fluid at region (1) accelerates to u_1+u' at point (2) or

$u_2 = u_1 + u'$

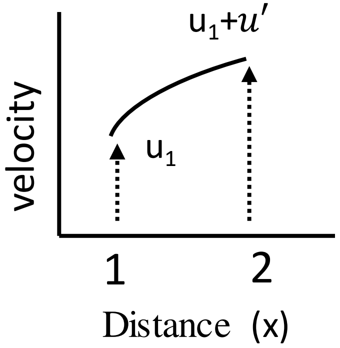
❖ **A)** Calculation of mixing length (λ_E):

The velocities at 1 and 2 can be related by :

$$u_2 = u_1 + \frac{du}{dx}(x_2 - x_1)$$

$$u_2 = u_1 + u'$$

$(x_2 - x_1) = \lambda_E$ Prandtl mixing length.



Velocity profile for turbulent flow

❖ **B)** Calculation of shear stress:

From the velocity profile

$$u_2 = u_1 + \lambda_E \frac{du}{dx} = u_1 + u'$$

$$u' = \lambda_E \frac{du}{dx}$$

where $u' = u_E$: mean eddy velocity or fluctuation velocity

$R = \rho u'^2$

$R = \rho \lambda_E^2 (\frac{du}{dx})^2$

E: eddy diffusivity $= \lambda_E^2 \parallel \frac{du}{dx} \parallel$

And rate of momentum due to eddy motion is

$R = E\rho \parallel \frac{du}{dx} \parallel$ for turbulent

the *eddy kinematic viscosity E,* the *eddy thermal diffusivity* E_H and the *eddy diffusivity* E_D analogous to *[*μ/ρ, $k / \rho C_p$ and D for molecular transport.

Extending the analogy further, E, E_H and E_D might be expected to be proportional to the product of a velocity term and a length term, each of which is characteristic of the eddies in the fluid. Whereas μ/ρ, $k / \rho C_p$ and D are all physical properties of the fluid and, for a material of given composition at a specified temperature and pressure have unique values, the eddy terms *E, EH* and *ED* all depend on the intensity of the eddies. In general, therefore, they are a function of the flow pattern and vary from point to point within the fluid.